## 2.9. Newton's Laws of Motion

In the previous articles on *Kinematics*, the different kinds of motions concerning the geometries of motions (i.e., positions etc. of these motions) have been considered without entering into the causes which produce these motions. The motions in classical mechanics concerning the cause and effect are governed by the *three laws of Newton*. These laws were enunciated by Newton in his 'Principia Mathematica' published in the year 1686.

### Newton's Laws of Motion

**First Law.** *Everybody continues in its state of rest or of uniform motion in a straight line, except in so far it is compelled by any external impressed force to change that state.*

**Second Law.** *The rate of change of momentum of a body is proportional to the impressed force, and takes place in the direction in which the force acts.*

**Third Law.** *To every action there is an equal and opposite reaction.*

The *first law* is also known as the '*Law of Inertia*'. The term inertia means the tendency of a body to continue as it is i.e., to remain in a state of rest or of uniform motion for ever in absence of any external force. This law gives us the qualitative definition of a force; i.e., a force is something which changes or tends to change the state of rest or of uniform motion of a body in a straight line.

The *second law* gives us a quantitative definition of a force i.e., it provides us with a measure of the applied force. The '*momentum*' of a body at any instant is defined as the product of its mass and the velocity at that instant.

**To deduce the formula : P = mf from the Second Law**

If $P$ be the external force which acting on a body of mass $m$ produces a velocity $v$ and acceleration $f$ at any instant $t$, then from the *first part* of the *second law* we have,

$$P \propto \frac{d}{dt}(mv), \quad \text{or,} \quad P = km \cdot \frac{dv}{dt},$$

where $m$ is constant with respect to $t$ and $k$ is the constant of variation.

$$\text{or,} \quad P = kmf, \tag{1}$$

where $\frac{dv}{dt}$ = acceleration $= f$.

If the unit of force be so chosen that it acting on unit mass produces unit acceleration, i.e., $P = 1$ when $m = 1$ and $f = 1$, then from (1), $1 = k.1.1$, or, $k = 1$.

Hence from (1), we have

$$P = \text{m}f. \tag{2}$$

COROLLARY. If $P = 0$ i.e., if there is *no impressed force*, then $\frac{d}{dt}(mv) = 0$ and hence $mv$ = constant, i.e., the body moves with constant momentum. In this case, the body moves with constant velocity.

OBSERVATIONS. Left-hand side of the equation (2) is known as the '*impressed force*' and the right-hand side as the '*effective force*'. It follows from equation (2) that if we apply forces in succession on the same mass and if they generate the same acceleration, then the forces must be equal. Again, if the same force be applied to two masses, and if it produces the same acceleration in them, then the masses must be equal. Thus, mass may be considered as the constant of proportionality between the impressed force and the produced acceleration. It also follows from equation (2) that $f = P/m$, i.e., *acceleration may be defined as the force per unit mass.*

Now from the second part of the law, we note that $P$ and $f$ have the same direction, i.e., an external force produces an acceleration in its direction. This is also generalised as, if two or more forces act on a body, each force being independent of other forces, produces an acceleration in its direction. This is known as *Law of Physical Independence of Forces*. Thus if a number of forces acting on a body produces equal number of accelerations in their respective directions, then equation (2) becomes

$$\Sigma\vec{P} = m\Sigma\vec{f}, \tag{3}$$

where the left hand side of equation (3) i.e., $\Sigma\vec{P}$ is the 'resultant impressed force' and the right-hand side is the 'resultant effective force'. From (3), it follows that the direction of the resultant impressed force and the direction of the resultant acceleration are the same, since $\Sigma\vec{P}$ and $\Sigma\vec{f}$ are like vectors and $m$ is a scalar. Equation (2) is known as '*the equation of motion*'. For a particle moving along a line this equation may be written as

$$m\ddot{x} = \text{the algebraic sum of the forces along the line.}$$

By 'the algebraic sum of the forces' we mean the sum of the forces with proper signs, the sign being positive when a force is in the sense of $x$ increasing and negative when the force is opposite to it.

The third law of motion gives us the idea that forces never exist singly, but always appear in pairs. If one body exerts a force on another, the second also exerts an equal force in the opposite direction. The first of these two forces is called the 'action' and the second one the 'reaction'. It should be noted that the action and its reaction do not act on the same body.

**Units of Force**

In **F.P.S.** (i.e., Foot-Pound-Second) system, the unit of force is called a **Poundal** and it is that amount of force which acting on a mass of one pound produces in it an acceleration of one foot per second per second.

In **C.G.S.** (i.e., Cm.-Gm.-Second) system, the unit of force is called a **Dyne**, and it is that amount of force which acting on a mass of 1 gm. produces in it an acceleration of 1 cm./sec$^2$.

$$1 \text{ poundal} = 30.48 \times 453.6 \text{ dynes.}$$

In *M.K.S.* (i.e., Metre-kg-second) system, the absolute unit of force is called a *Newton*, and it is that amount of force which acting on a mass of 1 kg. produces in it an acceleration of 1 metre/sec$^2$.

**Weight**

*The* **Weight** *of a body is the force with which the earth attracts the body towards its centre.*

Due to the attraction of the earth, acceleration of a freely falling body towards the earth is $g$. If $W$ be the weight of a body of mass $m$, then by *the second law* $W = mg$ which always acts vertically downwards.

We shall now consider the motion in a straight line under the action of *variable forces*. Let us first discuss the motion in a straight line under the action of *constant forces*.

❑ EXAMPLE 1. *A mass of* 10 *gm. falls freely from rest through* 10 *metres and is then brought to rest after penetrating* 5 *cm. of sand. Find the constant resistance of the sand in gm. weight.*

[C.U. B.A./B.Sc.]

SOLUTION. If $v$ cm./sec. be the velocity just before entering into sand, then

$$v^2 = 0^2 + 2g.1000 \quad [\because \; 10 \text{ metres} = 1000 \text{ cm.}]$$
$$= 2000g.$$

Let $R$ dynes be the constant upward resistance of sand on the mass of 10 gm. when it is penetrating into the sand, and due to this resistance, let $f$ be the retardation of the mass. Then since the mass is brought to rest after penetrating 5 cm. of sand, we have

$$0^2 = v^2 - 2f.5, \quad \text{or,} \quad 10f = v^2 = 2000g.$$
$$\therefore \; f = 200g. \text{ cm./sec}^2.$$

The resultant force acting on the mass of 10 gm. in the upward direction is $(R-10g)$ dynes. By *Newton's Second law*, we have

$$R-10g = 10f, \quad \text{where} \quad f = 200g \text{ cm./sec.}^2$$

or, $$R = 10g + 10f = 10(g+f) = 10(g+200g) = 2010g \text{ dynes} = \mathbf{2010 \text{ gm. wt.}}$$